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Allocating operating funding in the public sector and the newsvendor problem

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Public sector managers, particularly those at the highest level of government, tend to view lapsed (or unused) funds at the end of a fiscal year as a consequence of poor management and/or inadequate financial controls. The aim of this paper is to challenge this view. We show that the planning environment in the public sector is in essence the classical Newsvendor Problem. This simple model argues that lapsed funds are a direct consequence of a manager doing his job properly; that is, lapsed funds occur from time to time when a manager is maximizing value for the organization. An extension of the model shows that allowing low-value year-end spending has an undesirable effect on the value of spending during the year and this suggests a role for a strong audit function for year-end spending.

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1. Introduction

Public sector managers and those charged with their oversight tend to view lapsed funds (funds unused at the end of a fiscal year) a consequence of poor management and/or inadequate financial controls. This 'use it or lose it' culture is typically reinforced at the highest levels of government. For instance, in Canada in recent years, the Auditor General (2009) has criticized the Department of National Defence (DND) for allowing operating funds to lapse.

Our main purpose in this paper is to challenge this logic. We argue that the planning environment in a government department like DND can be modelled as a collection of planners whose individual decisions correspond to the classical Newsvendor Problem (NVP). The implication of this approach is that lapsed funds are the direct consequence of managers doing their jobs properly; that is, lapsed funds may occur as a matter of course when managers are maximizing value.

The NVP arises in a number of operations management and operations research applications. Typically a decision-maker is trying to determine how much of a particular good to acquire or produce in a situation where demand is uncertain and he must make this irrevocable decision before demand is realized. Generally, the decision-maker is trying to trade off the costs of too much supply *versus* too little. The interested reader is referred to Khouja (1999), Petruzzi and Dada (1999), and Silver *et al* (1998) for good reviews of the NVP literature.

The kernel of our NVP analogy for government departments posits a similar characteristic. We conceive a department's activities as comprising *Core* activities (high-value activities) and *Non-Core* activities (not essential but nonetheless valuable). The aggregate cost of Core activities is assumed to be uncertain. Moreover, we assume that decisions about Non-Core activities must be made before the aggregate cost of Core activities is known with certainty. In the case where our planner is given a fixed budget for the year, he/she has to balance the costs of not spending enough on Non-Core activities (and thus ending up with lapsed funds at year-end) with spending too much (some Core activities must be suspended in order to stay within the budget). This is, in essence, the same as the NVP.

The traditional approaches to the allocation of funds to projects (or activities) assume that the selected portfolio can be any subset of the list of projects that satisfies a budget constraint. The general objective is usually to maximize total portfolio utility or value subject to a budget constraint. Other public sector models use a multi-objective programming approach. For example, Greenberg and Nunamaker (1994) consider multiple goals and multiple restricted funding sources combined with the Analytic Hierarchy Process (AHP) that subjectively estimates selected components of their multi-objective budgeting model. A weighting method is then used to generate alternative feasible budgets to assist the decision-maker in selecting the final allocation of funds. Gabriel *et al* (2006) develop a similar multi-objective approach that uses AHP to determine the subjective ranking of the projects. They also include a probabilistic component that measures the risk of exceeding expected project costs, and in turn, the risk of

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exceeding the budget. Medaglia *et al* (2008) develop a mixed integer program to maximize a weighted sum of economic and financial net present values and a social impact index. Their model finds Pareto-optimal solutions that also include the best timings of the selected projects. Specific applications in the area of capital investment on defence projects include the work of Brown *et al* (1991), Brown *et al* (1994), Brown *et al* (2003), Loerch *et al* (1999), Ewing *et al* (2006), and Hurley and Schobel (2009). Hurley *et al* (2013) examine a similar structure of Core and Non-Core activities as we do here, but again the objective is to find an optimal portfolio of activities that have previously been ranked in order of importance and have been assigned individual values. Their model also allows the decision-maker to study the trade-off between the probability of lapsed funds and the probability of overspending.

All of these models are essentially *tactical/operational* models. Solving them will guide a decision-maker who actually has to make these project choices. But our purpose is different. We propose variants of the NVP that offer guidance on decisions at the *strategic* level. More specifically, they offer insight on lapsed funding, the nature of government department costs, and the incentives that should govern government department behaviour at the highest managerial levels. As such, the outcomes of the model are only important to the extent they shed light on these strategic issues. In other words, no manager would ever solve our models on a regular basis. They are meant only to guide our thinking on the nature of lapsed funding.

In summary, this paper presents a simple analytic model with the main purpose of better understanding the dynamics of lapsed funding in a government setting. The idea of using simple models to explain complex processes has several proponents in the literature. Geoffrion (1976) argues in a seminal work that such an approach is needed to gain insights into mathematical programming models. Hall (1986) supports this view by reasoning that simple models are easily understood and interpreted, and hence, useful. But these same authors also caution that such models are not meant in any way to replace or even compete with detailed mathematical programming models in the decision-making process. We apply the same caution here.

2. The idea of a newsvendor cost structure

We conceive the planning hierarchy in a government department as a collection of independent decision-makers (newsvendors). Each is given a budget and each allocates this budget to a set of *activities*. For instance in DND, activities are undertakings such as ‘Execute Operation A’, ‘Offer training course B’, ‘Transport public officials’, etc. These activities have three important characteristics:

1. *Differential value*. Some activities have higher values than others. For instance, a military operation would typically have a higher value than, say, the transport of public officials. In reality we conceive that there is a rank-order of importance in the activities. When put in rank-order, we assume there is a relatively continuous distribution of values. But for modelling purposes, we differentiate two kinds of activities, *Core* activities (or *Programmed* activities) and *Non-Core* activities (or *Non-Programmed* activities). Core activities are the most valuable activities and, for the most part, must be completed except in the case where DND is running out of funding towards the end of the year and must limit these activities. For example, if the plan called for 2000 Training Days on Ships, and DND had to limit spending at year-end, the Navy might choose to do fewer days to save money. So Core activities are required activities to the extent that DND must make some expenditure on them, but it is not necessary to complete an entire Core activity according to the budget plan in place at the start of the year. This curtailing of activities (either Core or Non-Core) towards the end of the year to save money is termed ‘off-ramping’.

   Non-Core activities are non-essential activities and less valuable than Core activities. In a subsequent variation of our main model, we’ll introduce *End-of-Year* activities in addition to Core and Non-Core activities. These are activities with relatively small positive values that can be executed towards the end of the year if there are end-of-year funds available to spend.
2. *Uncertain cost*. For some Core and Non-Core activities, it is not possible to determine the actual cost at the time the decision is made to undertake the activity. For most activities, an actual cost will only be known after the activity has been completed.
3. *Non-sequential execution*. It would be nice if the activities could be performed sequentially in order of their value. As each activity was completed, a decision-maker would know how much the completed activities had cost and therefore what was left to spend on the remaining activities. But of course this is not possible. Activities that add value to the organization must be done concurrently for a variety of good reasons. Thus, we assume there is considerable time overlap in the execution of the activities, and a decision-maker must make commitments to some lower-valued activities before he/she knows what some higher-value activities will actually cost.

These three factors in combination give rise to what we term a *Newsvendor Cost Structure* or *NV Cost Structure*.

Definition Suppose a government department is considering a set of Core and Non-Core activities for the upcoming fiscal year. Without loss in generality, we assume that the budget is just sufficient to cover the Core activity expenditures in the worst case. The department is said to have an *NV Cost Structure* if an irrevocable decision to go ahead with any Non-Core activities must be made before the cost of the Core activities is known with certainty.

Thus, the financial planner is faced with a dilemma. If only the Core activities are implemented, there is a positive probability that the budget will be underspent and lapsed funds will occur. On the other hand, by adding Non-Core activities, there is a positive probability that the budget will be overspent and off-ramping will be required.

To our knowledge, no other research has characterized a government department production function as proposed here. We believe this framework provides an important step forward in understanding the issue of lapsed departmental spending.

3. The newsvendor problem

In the face of uncertain demand, a newsvendor must make a one-time decision about how many papers to stock for the day. If he orders too few, he forgoes revenue and may upset those customers who find the shelf empty; if he orders too many, he can't sell them all and must take a loss, one whose size depends on what his supplier will give him for returned papers. His difficulty is that he must decide how many papers to carry before he knows what the demand is.

Mathematically, let D be a random variable representing uncertain daily demand. Suppose the paper costs the vendor c dollars per unit to procure, and he sells it at p dollars per unit. At the end of the day, unsold copies are returned to the supplier who reimburses the vendor c_s dollars per unit. Let us suppose that the vendor's beliefs about demand are characterized by a cumulative probability distribution $F(d) = Pr(D \leqslant d)$. We assume that the vendor will choose an order quantity x to maximize expected daily profit. This optimal value may be found using a marginal analysis.

The expected daily marginal gain accruing to the vendor by increasing his order size from a given number of units x_0 to $x_0 + 1$ would be

$$E[G] = (p-c)Pr\{D>x_0\} - (c-c_S)Pr\{D\leqslant x_0\}. \qquad (1)$$

As long as $E[G] > 0$, the rational vendor will increase the order size x, since this will improve his average daily profit. Thus, the rational vendor will increase x until $E[G]$ is just 0. This gives

$$(p-c)(1-F(x)) - (c-c_S)F(x) = 0, \qquad (2)$$

and solving, we get the classical condition

$$F(x^*) = \frac{p-c}{p-c_S} \qquad (3)$$

for the optimal order quantity x^*. It is important to note that, generally, it is not optimal for the vendor to order a quantity x equal to the expected demand $E(D)$. In the case where the profit from selling an additional newspaper, $p-c$, is less than the net cost of an unsold paper, $c-c_s$, and assuming a symmetric distribution for D the vendor would choose $x^* < E(D)$ and would be short most days.

4. A model for allocating operating funds in a government department

4.1. A single newsvendor

Consider a public sector manager and his decisions about how to allocate his operating funding to Core and Non-Core activities. What makes the problem difficult for the planner is the uncertain costs of the Core activities. The planner is assumed to have a fixed budget to fund both kinds of activities and any funds left over at the end of the year cannot be carried over to the next year. His problem is that he must make an irrevocable budget allocation to the Non-Core activities before he knows what the actual costs of the Core activities will be. So how should our planner allocate his funding?

The key observation linking this problem to the NVP is that the funds available for Non-Core activities may be viewed as an uncertain demand. (We could just as easily say an uncertain supply but we want to maintain the analogy with the NVP and its storied uncertain demand for newspapers.) Furthermore, the decision on how many Non-Core dollars to 'order' must be made before the 'demand' for these Non-Core dollars is known. As we argue below, this gives rise to an NV Cost Structure.

Consider now a planner with a budget B that cannot be exceeded. We let the random variable Q denote the expenditures by year-end on the Core activities and define

$$U = B - Q \qquad (4)$$

to be the funds available for the Non-Core activities (also termed slippage). Since budgets are normally set to ensure that all Core activities can be completed, we assume without loss in generality that $U \geqslant 0$. As with the NVP, the planner would have some idea of the probability distribution for U which is similarly modelled as some continuous cumulative distribution function $F_U(u)$ with $u \geqslant 0$. Thus it follows that if the manager does not allocate some budget to Non-Core activities, he will finish the year with lapsed funds with certainty. On the other hand, when any Non-Core activities are undertaken, there is a probability greater than zero that the budget will be overspent.

As noted above, the expenditure decision on Non-Core activities must be made before U is known with certainty. In reality a series of such decisions is made during the course of the year and the decision-maker may be able to use partial information on U at these points in time. Even so, most, if not all Non-Core activities must be implemented well before year-end in order to be completed during the budget cycle, and thus, much of the uncertainty in Core activity expenditures remains. For our purposes, we thus model the Non-Core expenditure decisions as a single variable x representing a single decision made at the start of the year. If it turns out that $U > x$, lapsed funds will occur. On the other hand, if $U < x$, the planner will exceed the budget B. To deal with the latter case, the manager must reduce ongoing activities at year-end (off-ramp) by the amount $x - U$. In sum, the assumptions are consistent with an NV Cost Structure.

Now we conduct a marginal analysis in the same way we did for the NVP. First we define v_C and v_{NC} to be the values accruing from each dollar spent on the Core and Non-Core activities respectively, and v_R to be the value lost per unit dollar reclaimed from off-ramped activities. If off-ramping is required at year end, the planner will shut down any ongoing activities in order of increasing value, starting with the lowest. However, in general, he will be required to shut down some combination of Core and Non-Core activities. There may also be additional shutting down costs in the form of penalties or intangibles that have the effect of reducing the off-ramp savings, or effectively increasing the value lost per net dollar of off-ramp. Thus, in general

$$v_{NC} < v_R, v_{NC} < v_C, \tag{5}$$

while the relationship between v_R and v_C cannot be fixed. If the costs to shut down are perceived to be high by the planner, it is possible for $v_R \geqslant v_C$.

In going from level x_0 to $x_0 + 1$, there is a marginal gain of v_{NC} if the demand is there and a net marginal loss of $v_R - v_{NC}$ if it is not. Thus the expected marginal gain would be

$$E[G] = v_{NC}(1 - F_U(x_0)) - (v_R - v_{NC})F_U(x_0). \tag{6}$$

Setting this to 0 and solving gives

$$F_U(x^*) = \frac{v_{NC}}{v_R} \tag{7}$$

where x^* is the optimal level of Non-Core expenditure. For the purposes of this paper we make the following definition:

Definition x^* is the *efficient* level of Non-Core expenditure.

As with the NVP, the financial planner will rarely if ever get it right. Faced with uncertain demand for Non-Core activity dollars and the imperative to decide at the beginning of the year how many of these dollars to procure, the planner will inevitably be left with either an under-spent or over-spent budget towards the end of the year. Defining the value ratio,

$$r = \frac{v_{NC}}{v_R} < 1, \tag{8}$$

the respective probabilities are

$$Pr\{\text{Lapsed Funds}\} = Pr(U > x^*) = 1 - F_U(x^*) = 1 - r \tag{9}$$

and

$$Pr\{\text{Off} - \text{Ramp}\} = Pr(U < x^*) = F_U(x^*) = r. \tag{10}$$

Typically the number of planned Core activities is quite large; and many of them are carried out independently of one another. Thus, under reasonable assumptions on the probability distributions of their costs, it follows by Lyapunov's Central Limit Theorem (see eg Billingsley, 1996) that total slippage U behaves, at least in approximation, as a normal random variable. Letting μ_U and σ_U denote, respectively, the mean and standard deviation of U, and $\Phi(z)$ the cumulative distribution function of the standard normal, we obtain under the assumption of normality that

$$x^* = \mu_U + z^* \sigma_U, \tag{11}$$

where

$$z^* = \Phi^{-1}(r) \tag{12}$$

is obtained directly from the standard normal table for the specified value of r. If $r < 1/2$, then z^* will be negative and $x^* < \mu_U$; if $r > 1/2$, then z^* will be positive and $x^* > \mu_U$.

4.2. A department as a collection of newsvendors

There are literally hundreds of budget-holders within DND. Although they are organized in a hierarchy, we consider the case where financial decision-making is equivalent to m independent newsvendors. This is reasonable in the case where the DND hierarchy can be thought of as a network with a tree structure. The m independent newsvendors, then, are the end-nodes of this tree. The upper structure of the tree is there simply to allocate funds to these end-nodes. Once in possession of a budget allocation, each end-node allocates its budget among Core and Non-Core activities.

Suppose end-node i (EN i) has uncertain funds available for its Non-Core activities given by U_i. Let EN i's value ratio be r_i and let his optimal expenditure on Non-Core activities, x_i^*, satisfy

$$F_{U_i}(x_i^*) = r_i. \tag{13}$$

We assume that the U_i are independent random variables. The lapsed funding for EN i is

$$L_i = \max(U_i - x_i^*, 0); \tag{14}$$

off-ramp spending is

$$O_i = \max(x_i^* - U_i, 0). \tag{15}$$

Total lapsed funding for the organization is

$$L = \sum_i L_i \tag{16}$$

and off-ramp spending is

$$O = \sum_i O_i. \tag{17}$$

We consider two regimes for the management of these end-nodes. In one, termed the *Shadow Banking Regime*, those ENs with lapsed funds can transfer their excess funds to ENs who are planning to take off-ramps. The process of transferring these funds would be controlled by the upper levels of the hierarchy. It is a sophisticated, delicate system that requires significant oversight and managerial skill. There is a related literature coming out of the NVP. The interested reader is referred to Slikker *et al* (2005) and Hanany *et al* (2010). These papers look at a cooperative game among decentralized newsvendors when there is transshipment. Although these models are similar, they are not quite the same. For instance, consider the Slikker *et al* (2005) paper. They ask whether it makes sense for newsvendors to cooperate when placing orders to a manufacturer. In our model, that would be equivalent to government departments sitting down at the start of the year to fix their joint Non-core spending. This might be a sensible thing to do for related departments but not for all. This difference will become clear with the development below.

At the other extreme, we have the *No Shadow Banking Regime* where no exchanges are allowed among ENs. Under this set of rules, the probability of observing positive lapsed funding across the organization is

$$\begin{aligned} Pr\{\text{Lapsed Funds}\} &= Pr\{L>0\} \\ &= 1 - Pr\{U_i \leqslant x_i^*,\ i = 1,2,\ldots,m\} \\ &= 1 - \prod_{i=1}^{m} Pr\{U_i \leqslant x_i^*\} \\ &= 1 - \prod_{i=1}^{m} r_i. \end{aligned} \tag{18}$$

The equality in the third line above follows from the independence of the U_i; that is, since there are no interactions among ENs, we have essentially m independent single NVPs.

Assuming the same value ratio for all ENs, $r_i = r$ for $i = 1, 2, \ldots, m$, we obtain the simple relation

$$Pr\{L>0\} = 1 - r^m. \tag{19}$$

Suppose $r = 0.95$ and $m = 100$. Then $\Pr\{L>0\} = 0.995$. Thus, even with a high value ratio, lapsed funding will be observed with near certainty.

Consider next the case of the Shadow Banking Regime where those ENs with lapsed funds may have their surpluses transferred to those ENs with planned off-ramps. Let us assume for now that ENs make the same decision on Non-Core activities as in the No Shadow Banking Regime; that is, they use their efficient level of Non-Core spending as defined in (7). We will have more to say about this later.

Under this system, if $L > O$, no off-ramping will be required, and lapsed funding in the amount $L - O$ is observed across the organization. If $L = O$ the budget is balanced, while $L < O$ implies the total budget has been exceeded and some off-ramping must occur.

Letting $T = L - O$ the probability of lapsed funding for the organization becomes

$$Pr\{T>0\} = Pr\left\{\sum_{i=1}^{m} (U_i - x_i^*) > 0\right\}. \tag{20}$$

Assuming that each U_i is normally distributed with mean μ_i and variance σ_i^2, as discussed above, we have

$$U_i - x_i^* = U_i - (\mu_i + z_i^* \sigma_i), \tag{21}$$

where $z_i^* = \Phi^{-1}(r_i)$, $i = 1, 2, \ldots, m$, so that

$$Pr\{T>0\} = Pr\left\{\sum_{i=1}^{m} U_i > \sum_{i=1}^{m} (\mu_i + z_i^* \sigma_i)\right\}. \tag{22}$$

To simplify further, we use the same value ratio, $r_i = r$, for all units, and also assume that the standard deviation is constant across all ENs, so that $\sigma_i = \sigma$, $i = 1, 2, \ldots, m$. The total slippage,

$$U_T = \sum_{i=1}^{m} U_i, \tag{23}$$

is then normally distributed with mean

$$\mu_T = \sum_{i=1}^{m} \mu_i \tag{24}$$

and variance

$$\sigma_T^2 = \sum_{i=1}^{m} \sigma_i^2 = m\sigma^2. \tag{25}$$

Hence we have that

$$\begin{aligned} Pr\{T>0\} &= Pr\left\{\sum_{i=1}^{m} U_i > \sum_{i=1}^{m} (\mu_i + z_i^* \sigma_i)\right\} \\ &= Pr\{U_T > \mu_T + z^* m\sigma\} \\ &= Pr\left\{Z = \frac{U_T - \mu_T}{\sigma_T} > \sqrt{m} z^*\right\} \end{aligned} \tag{26}$$

where $z^* = \Phi^{-1}(r)$. This gives

$$Pr\{T>0\} = 1 - \Phi\left(\sqrt{m} z^*\right) = 1 - \Phi\left(\sqrt{m}\,\Phi^{-1}(r)\right). \tag{27}$$

Values of $\Pr\{T>0\}$ for various values of r and m are shown in Table 1. Note that the probability of lapsed funds approaches 1 quickly for large m, and r decreasing below 0.50. Hence, even with a perfect Shadow Banking System, there is a wide range of values of r that leads to organization-wide lapsed funds with near certainty.

Let us now return to the assumption that ENs sharing funds towards the end of the fiscal year has no affect on initial Non-Core expenditures. If an efficient system is available for transferring funds between departments at year end, intuitively one would expect managers to commit more dollars to Non-Core activities at the start of the year. If it turns out that the budget is exceeded in a department, the manager would anticipate relief from other departments where it happens that underspending occurred. We may show in fact that in an idealized game with a large number of independent and

Table 1 Probability of lapsed funding for various values of r and m

	Pr{Lapsed Funds}			
r	$m=25$	$m=50$	$m=100$	$m=150$
0.50	0.500	0.500	0.500	0.500
0.49	0.550	0.895	0.994	1.000
0.48	0.599	0.994	1.000	1.000
0.47	0.647	0.999	1.000	1.000
0.46	0.692	1.000	1.000	1.000
0.45	0.735	1.000	1.000	1.000

identical players ($m \to \infty$), and where we assume transshipment costs (using the terminology of Slikker *et al*, 2005) are zero, equilibrium will be reached when each player commits an amount of Non-Core spending equal to the mean of the distribution of U ($x^* = \mu$). In this scenario, the proportion of lapsed funds is driven to zero!

Unfortunately, the actual banking system in play within DND is somewhere between the two extremes discussed, and probably closest to the No Shadow Banking regime. There are several factors that work against an idealized free-flowing system among all ENs. We list a few important ones below:

1. Generally, there is only a loose central control mechanism to administer the transfer of funds between departments. Such transfers occur informally between departments with close ties, for example manager A knows manager B. The effect is to seriously limit the number of players in the game.
2. The availability of year-end spending (discussed in the next sub-section) reduces the funds that would otherwise be available to other departments, since this type of spending adds value to the unit.
3. Managers must initiate off-ramps before it is too late, and sometimes before the availability of transfer funds is completely known

We finally note that the Newsvendor framework is still useful in explaining managerial behaviour when an imperfect banking system is in place. In essence, the manager's perception of the coefficient v_R changes; that is, a lower value is perceived when some transfer funds may become available at year-end. Equivalently, the distribution of U may be adjusted according to the manager's assessment on the availability of these transfer funds.

4.3. The hidden effect of year-end spending

We now introduce a third kind of spending, year-end spending. Leibman and Mahoney (2010) do a sophisticated econometric measurement of the quality of year-end spending for federal government departments in the US. They begin by documenting the pattern of departmental discretionary expenditures and find that there is a huge surge in spending in the last week of the fiscal year. This spending is concentrated in construction-related goods, furnishings and equipment, and IT equipment and services. To get at the quality of this spending, they focussed on IT expenditures. Based on a data set of 686 major IT projects (130 billion dollars in cost), spending on these projects in the last week of the year is observed to be about seven times the average amount across the other 51 weeks. More importantly they are able to show that the quality of the week 52 expenditure is lower:

'In tandem with the spending increase, there is a sharp drop-off in investment quality. Based on a categorical index of overall investment performance, which combines assessments from agency information officers with data on cost and timeliness, we find that projects that originate in the last week of the fiscal year have 2.2 to 5.6 times higher odds of having a lower quality score. Ordered logit and OLS regressions show that this effect is robust to agency and year specific factors as well as to a rich set of project characteristic controls'.[1]

The Liebman and Mahoney measurement constitutes significant empirical support for the position that US government year-end spending is of lower quality. Whether the same conclusions would be found in Canada is another matter. However, Canadian government year-end discretionary spending tends to be for the same classes of goods and services; so we expect the same result would hold.

So suppose that, at year-end, the planner may reduce any lapsed funds by spending on last-minute activities or items of unit value v_S. These activities (items) by their very nature tend to contribute the least value. Following the empirical discussion above, we assume that $v_S > 0$ and $v_S < v_{NC}$.

Now the net marginal gain for each additional dollar allocated earlier in the year for Non-Core activities needs to be adjusted from v_{NC} to $v_{NC} - v_S$, since the do-nothing strategy will be rewarded at year-end by unit value spending of v_S. Thus replacing x by y for this new scenario, the expected marginal gain from the Non-Core activity decision becomes:

$$E[G] = (v_{NC} - v_S)(1 - F_U(y)) - (v_R - v_{NC})F_U(y). \qquad (28)$$

Setting this expression to 0 yields an adjusted value ratio for finding the optimal order quantity, y^*:

$$F_U(y^*) = \frac{v_{NC} - v_S}{v_R - v_S} = r'. \qquad (29)$$

Since $v_S < v_{NC} < v_R$ it is clear that $r' \in (0, 1)$. We also see how year-end spending affects the planner's Non-Core decision. If $v_S \to v_{NC}$, $r' \to 0$ so that the planner delays entirely the decision to overprogram to the end. However, if $v_S \to 0$, $r' \to v_{NC}/v_R = r$ and the planner's Non-Core decision at the beginning of the year is unchanged ($y^* = x^*$). Still, the following general result holds as long as $v_S > 0$.

Property $y^* < x^*$.

Proof $r' = \alpha r$ where

$$\alpha = \frac{1 - v_S/v_{NC}}{1 - v_S/v_R} < 1. \qquad (30)$$

Thus,

$$r' < r \Rightarrow F_U(y^*) < F_U(x^*) \Rightarrow y^* < x^*. \qquad \square$$

Thus, the manager who has unlimited ability to spend funds of positive value v_S at year-end *will commit fewer funds to Non-Core activities at the start of the year*. Equivalently, the

[1]See Leibman and Mahoney (2010, p 3). The explicit details of the measurement are included later in their paper.

manager will choose an *inefficient* level of Non-Core expenditure. In real life, however, the manager does not have unlimited ability for value-added year-end spending. First, there may be very few activities available that can be planned, implemented and completed in the short time frame allowed. Second, the manager will have only a restricted number of hours left available for his harried staff to implement and supervise these last-minute activities. Nevertheless, the basic conclusion—that the manager will behave more conservatively during the year—remains.

4.4. Summary of our results

In our model, we are able to assess the actions a planner would take under two incentives. In one, termed *Value Maximization*, we assume that he/she will maximize the aggregate value of the projects adopted; in the other, termed *Zero Lapsed Funds*, he/she does not care about value maximization, but rather making sure all of the available budget is expended. Our model suggests a number of important results:

Result 1 Under the assumption that a department has an NV Cost Structure and managers have a Value Maximization incentive, we show that the selection of the spending level on Non-Core activities is equivalent to a Newsvendor Problem.

Result 2 Under the assumption of an NV Cost Structure and the Value Maximization incentive, the optimal choice of spending level on Non-Core activities (which we define to be the *efficient* outcome) will give rise to lapsed funds some of the time. Hence lapsed funding is a by-product of a manager doing his job properly.

Result 3 Under the assumption of an NV Cost Structure and that managers have a Zero Lapsed Funds incentive, a manager chooses a spending level on Non-Core activities that guarantees 0 lapsed funds but is also inefficient (ie the spending level results in a lower expected valuation than under the Value Maximization incentive). We see this immediately from Equation (6). Under the Zero Lapsed Funds incentive, $F_U(x_0) \to 1$ and thus, the expected marginal gain takes on a negative value ($E[G] < 0$).

We expand our model with a third type of spending, end-of-year spending, as defined above. With the three types of spending and the Value Maximization incentive, we show the following result.

Result 4 Under the assumption of an NV Cost Structure and Value Maximization, the existence of end-of-year activities to soak up spare end-of-year dollars that would otherwise lapse results in an inefficient choice for Non-Core activities, and hence, the net value added from Non-Core spending is lower than it otherwise would be.

5. Applying the model

5.1. Responding to the Auditor General

Our point of departure is the recent public rebuke of DND by the Auditor General. In fiscal 2007–2008, DND incurred lapsed operational funding in excess of \$300 million, an amount that was just over 2% of DND's 2007–2008 spending authority (or budget). In response to this outcome, the Auditor General commented:

> The lack of accurate and timely information for decision makers contributed to the lapsing of more than \$300 million in funding that was available during the 2007–08 fiscal year but is now permanently unavailable to National Defence.[2]

The implication is that lapsed funding is not a good thing.

Which incentive then—Value Maximization or Zero Lapsed Funds—governs actual DND manager behaviour. High level criticism by the Auditor General and within DND certainly pushes managers to favour the Zero Lapsed Funds incentive. On the other hand, under the assumption of an NV Cost Structure, our model suggests that the most efficient outcome occurs when there is Value Maximization and, with this incentive and cost structure, lapsed funds are bound to occur. In other words, lapsed funds may be interpreted as a sign that a manager is doing his job properly. So what is the evidence on lapsed funding?

We present the total budget authorities, actual spending and lapsed funding for DND operations for the period 2002–2010 in Table 2. In two of the last three years, the lapsed funding has exceeded \$400 million. But these amounts do not represent permanent lapsed funding. For example, take the lapsed funding in 2009–2010. An inspection of the DND Performance Report for 2010 reveals that, taking into account various carryover provisions, only \$123.4 million of the lapsed funding became permanently unavailable to the Department. The same check for 2008–2009 suggests that only \$31.9 million was permanently lost. In this sense, the lapsed funding reported in the Public Accounts (Table 2) is an upper bound on permanent lapsed funding.

Based on our theory of the DND organization as a set of newsvendors, there is a wide range in the value ratio, *r*, that will give rise to aggregate lapsed funding every year. We might thus conclude that some value maximization is occurring within the organization.

In summary, we see that under the NV cost structure, a policy of zero lapsed funds will result in an inefficient allocation of Non-Core spending. The manager will maximize spending on Non-Core activities, leading inevitably to a costly off-ramp at the end of the year. On the other hand, a policy of value maximization where year-end spending is allowed is also shown by a simple extension of the model to lead to inefficient

[2]See Chapter 5 of the 2009 Spring Report of the Auditor General of Canada available at http://www.oag-bvg.gc.ca/internet/English/parl_oag_200905_e_32545.html.

Table 2 Vote 1 lapsed funding over the period 2002–2010 (all values in thousands of Canadian dollars)

Fiscal year	*Total authorities*	*Actual spending*	*Lapsed funding*
2009–2010	15 204 236	14 792 353	411 883
2008–2009	14 381 794	14 283 787	98 007
2007–2008	13 234 228	12 812 313	421 915
2006–2007	12 014 953	11 925 234	89 719
2005–2006	11 107 947	11 093 092	14 855
2004–2005	10 669 994	10 474 202	195 792
2003–2004	10 120 800	9 867 900	252 900
2002–2003	9 394 600	9 319 700	74 900

Non-Core spending. The implication that a better information system will eliminate the problem of lapsed funds, as the Auditor General suggests, is simply wrong.

Leibman and Mahoney (2010) offer some interesting evidence on the worth of a carryforward provision. They make an interesting comparison between the US Department of Defence which has no carryforward provision and the US Department of Justice which can carryforward funds to the next year. Their empirical evidence suggests that, with the Department of Justice, there is a much smaller spike in year-end spending and that the quality of this year-end spending is not significantly lower than it is over the rest of the year whereas with the Department of Defence it is. This would suggest that a Value Maximization incentive coupled with a carryforward provision and a strong audit of year-end activity expenditures might be a very useful combination of incentives.

5.2. What level of lapsed funds is acceptable?

Consider the following scenario with two managers from different departments. Manager 1 averages \$4 million annually of unused funds. He may be doing a very poor job if Manager 2, in a similar environment and with the same budget, averages \$2 million in lapsed funds. In fact, Manager 1 could be adding less value to his unit than if he regularly overspent the budget and incurred a reasonable level of off-ramping at year-end; that is, by following a plan closer to the zero lapsed funds policy. On the other hand, we could argue that Manager 2 is incurring excessive levels of off-ramping. Which of the two is doing a better job?

Again, the NV model presented here is not meant to be used as an operational decision tool for managers. However, we believe it may still be useful for setting a limit (or goal) on an acceptable average level of lapsed funding. We illustrate this idea in the context of the example described above. Suppose a 'rational' manager who is allocating funds to Non-Core activities during the year is faced with an uncertain demand for Non-Core dollars in the range of 0 to \$10 million. The average amount of lapsed funds in the manager's hands at the end of the year is:

$$E[\text{lapsed funds}] = \int_{x^*}^{\Delta} (u - x^*) f_U(u)\,\mathrm{d}u, \tag{31}$$

where $\Delta = \$10$ million, $f_U(u)$ is the density function of U, and x^* satisfies (7). To continue the example, let us assume (conservatively) that U is a uniform random variable in the interval $[0, \Delta]$; that is, the density function

$$f_U(u) = \frac{1}{\Delta}, \quad 0 \leqslant u \leqslant \Delta. \tag{32}$$

Then (7) yields

$$x^* = r\Delta, \tag{33}$$

and (31) is readily shown to simplify to

$$E[\text{lapsed funds}] = (1 - r)^2 \frac{\Delta}{2}. \tag{34}$$

Suppose now that the value ratio $r = 1/3$; that is, one off-ramped unit is perceived to be three times more valuable than one added unit of Non-Core activity. The average amount of lapsed funds in this case should be

$$\left(1 - \frac{1}{3}\right)^2 \times \frac{\$\,10\,000\,000}{2} = \$\,2\,220\,000. \tag{35}$$

Based on this estimate, we would conclude that Manager 1 is not maximizing the long-term value of Non-Core spending. It may be that his aversion to the possibility of off-ramping is too high, or equivalently, his perception of the value ratio ($r \simeq 0.1$) is too low. Meanwhile Manager 2 appears to be doing a much better job of balancing the average values lost from lapsed funds and from off-ramping.

6. Conclusions

Our main point of departure in this paper is the critique of DND lapsed funding by the Auditor General of Canada. This critique is indicative of the conventional wisdom about government lapsed funding in general, the well known 'use it or lose it' advice to government managers. In our view, this public rebuke of DND demonstrates a poor understanding of the nature of DND spending and could lead to serious inefficiency depending on how strongly DND is influenced by the incentive to spend all authorized funding.

Our analysis turns on the concept of an NV Cost Structure. With this structure, we show that a government department's decisions on activity expenditures can be boiled down to an NVP. This characterization has a number of important implications for government fiscal management:

1. In the normal course, and assuming that government managers maximize the total value of Core and Non-Core activities, we can reasonably expect to see lapsed funding in most years.

2. The encouragement of public managers to use all available funds may lead to inefficiency. Thus our Newsvendor model supports the empirical results of Leibman and Mahoney (2010) demonstrating that the most efficient oversight regime is one where managers are able to carryforward lapsed funds up to some limit into the next fiscal year.

A future area of research is the extension of our basic model to a dynamic model with multiple decision points, different levels of uncertainty through the fiscal year and the incorporation of a carryforward provision. A formal process for setting value ratios also needs to be researched. Finally, a game-theoretic approach to investigate the effects of shadow banking on managerial behaviour is also required.

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